Adaptive Group-Blind Multiuser Detection Based on a New Subspace Tracking Algorithm

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***Abstract*—Recently, Wang and Høst-Madsen developed group-blind multiuser detectors for use in code-division multiple-access (CDMA) uplink environments in which the base station receiver has the knowledge of the spreading sequences of all the users within the cell, but not that of the users from other cells. Yu and Høst-Madsen later developed an adaptive version of this detector for synchronous CDMA channels. In this letter, we develop a new low-complexity, high-performance subspace tracking algorithm and apply it to adaptive group-blind multiuser detection in asynchronous multipath CDMA channels. The detector can track changes in the number of users and their composite signature waveforms. We present steady-state performance as well as the ability of the receiver to track changes in the signal subspace. We also address the performance gain of the group-blind detector over its blind counterpart for this application.**

***Index Terms*—Adaptive multiuser detection, code-division multiple access (CDMA) uplink, group-blind multiuser detection, subspace tracking.**

I. Introduction

BLIND multiuser detection using subspace techniques was first developed in depth by Wang and Poor [1], [2]. Such techniques were appropriate for downlink environments where only the desired users' code is available. More recently, these subspace techniques were extended by Wang and Høst-Madsen to uplink environments where the base station knows the codes of in-cell users, but not those of users outside the cell [3]. This new family of detectors has been termed group-blind multiuser detectors. One attractive member of this family, the group-blind linear hybrid detector, performs very well compared with the other group-blind detectors, even though it has the lowest computational complexity. In this letter, we develop a new, low-complexity, high-performance subspace tracking algorithm and use it, along with the closed-form expression for the hybrid group-blind detector, to develop an *adaptive* group-blind multiuser detector for slowly-varying asynchronous dispersive code-division multiple-access (CDMA) channels. We will also compare the performance of the group-blind detector to that of the blind detector that makes use only of the composite waveform of the user of interest.

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The rest of this letter is organized as follows. In Section II, we summarize the signal model. In Section III, we review subspace methods of blind and group-blind multiuser detection. In Section IV, we address the role of subspace tracking in our adaptive receiver and introduce a new low-complexity, high-performance subspace tracking algorithm. Simulation results are provided in Sections V, and Section VI concludes the letter.

II. Signal Model

We adopt the asynchronous multipath CDMA model developed by Wang and Høst-Madsen. In the interest of brevity, we summarize the model here and refer the reader to [3] for a full development. Consider a K-user binary communication system, employing normalized modulating waveforms $s_1, s_2, \ldots, s_k$, and signaling through their respective multipath channels with additive Gaussian noise. The transmitted signal due to the kth user is given by

$$x_k(t) = A_k \sum_{i=0}^{M-1} b_k[i] s_k(t - iT - d_k) \tag{1}$$

where M denotes the length of the data frame and T denotes the information symbol interval; A_k, $\{b_k[i]\}$, and $d_k \in [0, T)$ denote, respectively, the amplitude, symbol stream, and the delay of the kth user's signal. We assume that for each k, the symbol stream $\{b_k[i]\}$ is a collection of independent random variables that take on values of $+1$ and -1 with equal probability. Furthermore, we assume that the symbol streams of different users are independent. For the direct-sequence spread-spectrum (DS-SS) format, the user signaling waveforms have the form

$$s_k(t) = \sum_{j=0}^{N-1} c_k[j] \psi(t - jT_c), \qquad 0 \le t \le T \tag{2}$$

where N is the processing gain, $\{c_k[j]\}$ is a signature sequence of ± 1's assigned to the kth user, and $\psi(t)$ is a normalized chip waveform of duration $T_c = T/N$. The kth user's signal $x_k(t)$ propagates through a multipath channel whose impulse response is given by

$$g_k(t) = \sum_{l=1}^{L} \alpha_{kl}\, \delta(t - \tau_{kl}) \tag{3}$$

where L is the number of paths in the channel; α_{kl} and τ_{kl} are, respectively, the complex path gain and delay of the lth path of the kth user. It is assumed that the channel is slowly varying, so that the path gains and delays remain constant over the duration of one signal frame (MT). The received signal component due to the transmission of the kth user's signal through the channel $g_k(t)$ is given by

$$y_k(t) = x_k(t) \star g_k(t). \tag{4}$$

The total received signal at the base station receiver is the superposition of the K user's signals, plus additive Gaussian noise and is given by

$$r(t) = \sum_{k=1}^{K} y_k(t) + v(t) \tag{5}$$

where $v(t)$ is a zero-mean complex Gaussian noise process.

At the receiver, the received signal is match filtered (to the chip waveform) and sampled at a multiple (p) of the chip rate, i.e., the sampling interval is $\Delta = T_c/p = T/P$, where $P \triangleq pN$ is the total number of samples per symbol interval. The nth received signal sample during the ith symbol is given by

$$\begin{aligned} r[i, n] =& h_k[0, n]b_k[i] + \underbrace{\sum_{j=1}^{\iota_k} h_k[j, n]b_k[i-j]}_{\text{ISI}} \\ &+ \underbrace{\sum_{k' \neq k} \sum_{j=0}^{\iota_k} h_{k'}[j, n]b_k[i-j] + v[i, n]}_{\text{MAI}} \end{aligned} \tag{6}$$

where we denote

$$\iota_k \triangleq \left\lceil \frac{d_k + \tau_{kL} + T_c}{T} \right\rceil \tag{7}$$

$$h_k[i,n] \triangleq h_k(iT + n\Delta) \tag{8}$$

$$h_k(t) \triangleq A_k s_k(t - d_k) \star g_k(t). \tag{9}$$

In (6), the first term contains the ith bit of the kth user; the second term contains the intersymbol interference (ISI) from the previous bits of the kth user; the third term contains the multiple-access interference (MAI) from the other users; the last term is the ambient channel noise. Denote

$$\underline{r}[i] \triangleq \begin{bmatrix} r[i, 0] \\ \vdots \\ r[i, P-1] \end{bmatrix}_{P\times 1} \qquad \underline{v}[i] \triangleq \begin{bmatrix} v[i, 0] \\ \vdots \\ v[i, P-1] \end{bmatrix}_{P\times 1}$$

$$\underline{b}[i] \triangleq \begin{bmatrix} b_1[i] \\ \vdots \\ b_K[i] \end{bmatrix}_{K\times 1} \tag{10}$$

$$\underline{H}[j] \triangleq \begin{bmatrix} h_1[j, 0] & \cdots & h_K[j, 0] \\ \vdots & \vdots & \vdots \\ h_1[j, P-1] & \cdots & h_K[j, P-1] \end{bmatrix}_{P\times K}, \quad j = 0, 1, \ldots, \iota_k. \tag{11}$$

Then we may write

$$\underline{r}[i] = \underline{H}[i] \star \underline{b}[i] + \underline{v}[i]. \tag{12}$$

By stacking m successive received sample vectors, we can write (12) in matrix form as

$$\mathbf{r}[i] = \mathbf{H}\mathbf{b}[i] + \mathbf{v}[i]. \tag{13}$$

The smoothing factor m is chosen such that $m \geq \lceil (P+K)/(P-K) \rceil \cdot \max_k\{\iota_k\}$ for channel identifiability [3]. Note that the columns of $\mathbf{H}$ (the composite signature vectors) contain information about both the timings and the complex path gains of the multipath channel of each user. Hence, an estimate of these waveforms eliminates the need for separate estimates of the timing information $\{\tau_{kl}\}_{l=1}^{L}$.

III. Group-Blind Multiuser Detection

Since the ambient noise is white, i.e., $\mathrm{E}\{\mathbf{v}[i]\mathbf{v}[i]^H\} = \sigma^2\mathbf{I}$, the autocorrelation matrix of the received signal in (13) is

$$\mathbf{X} \triangleq \mathrm{E}\{\mathbf{r}[i]\mathbf{r}[i]^H\} = \mathbf{H}\mathbf{H}^H + \sigma^2\mathbf{I} \tag{14}$$

$$= \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{U}_s^H + \sigma^2\mathbf{U}_n\mathbf{U}_n^H \tag{15}$$

where (15) is the eigendecomposition of $\mathbf{X}$. In the group-blind multiuser detection scenario, we assume we have knowledge of the first $\tilde{K}$, $\tilde{K} \leq K$ users' spreading sequences, whereas the rest of the users are unknown to the receiver.

Define the set of matrices $\{\tilde{\mathbf{H}}_j\}_{j=0}^{m+\iota-1}$ such that $\tilde{\mathbf{H}}_j$ is the $Pm \times \tilde{K}$ matrix composed of columns $jK+1$ through $jK + \tilde{K}$ of the matrix $\mathbf{H}$. We define the matrix $\tilde{\mathbf{H}} \triangleq [\tilde{\mathbf{H}}_0\tilde{\mathbf{H}}_1 \cdots \tilde{\mathbf{H}}_{\iota+m-1}]$. The size of $\tilde{\mathbf{H}}$ is $Pm \times \tilde{r}$ where $\tilde{r} \triangleq \tilde{K}(m+\iota)$. Then the group-blind linear hybrid detector for user k, $k = 1, \ldots, \tilde{K}$, is given by the solution to the following constrained optimization problem:

$$\mathbf{w}_k = \arg \min_{\mathbf{w} \in \mathrm{range}(\mathbf{H})} E\left\{\left|b_k[i] - \mathbf{w}^H\mathbf{r}[i]\right|^2\right\} \tag{16}$$

subject to the constraint $\mathbf{w}^H\tilde{\mathbf{H}} = \mathbf{1}_{\tilde{K}\iota+k}^T$ where $\mathbf{1}_l$ is the vector of length $\tilde{K}(m+\iota)$ each of whose elements is zero, except the lth element which is 1. Heuristically speaking, this detector zero-forces the interference caused by the $\tilde{K}$ known users, and suppresses the interference from unknown users according to the minimum mean-square-error (MMSE) criterion. The solution (form II) and the corresponding bit estimate for user k may be written as [3]

$$\mathbf{w}_k = \mathbf{U}_s\mathbf{\Lambda}_s^{-1}\mathbf{U}_s^H\tilde{\mathbf{H}}[\tilde{\mathbf{H}}^H\mathbf{U}_s\mathbf{\Lambda}_s^{-1}\mathbf{U}_s^H\tilde{\mathbf{H}}]^{-1}\mathbf{1}_{\tilde{K}\iota+k} \tag{17}$$

$$\hat{b}_k[i] = \mathrm{sgn}\left\{\mathrm{Re}\left(\mathbf{w}_k^H\mathbf{r}[i]\right)\right\}, \qquad k = 1, 2, \ldots, \tilde{K}. \tag{18}$$

The linear *blind* MMSE multiuser detector, which assumes knowledge only of the signature waveform of the user of interest, is given by [1]

$$\mathbf{m}_k = \mathbf{U}_s\mathbf{\Lambda}_s^{-1}\mathbf{U}_s^H\tilde{\mathbf{H}}\mathbf{1}_{\tilde{K}\iota+k}. \tag{19}$$

Of course, we require knowledge of the composite signature waveforms of the first $\tilde{K}$ users in order to construct $\tilde{\mathbf{H}}$. To estimate these waveforms, we take advantage of the fact that the noise subspace is orthogonal to the column space of $\mathbf{H}$. However, computing the noise subspace increases complexity, so we

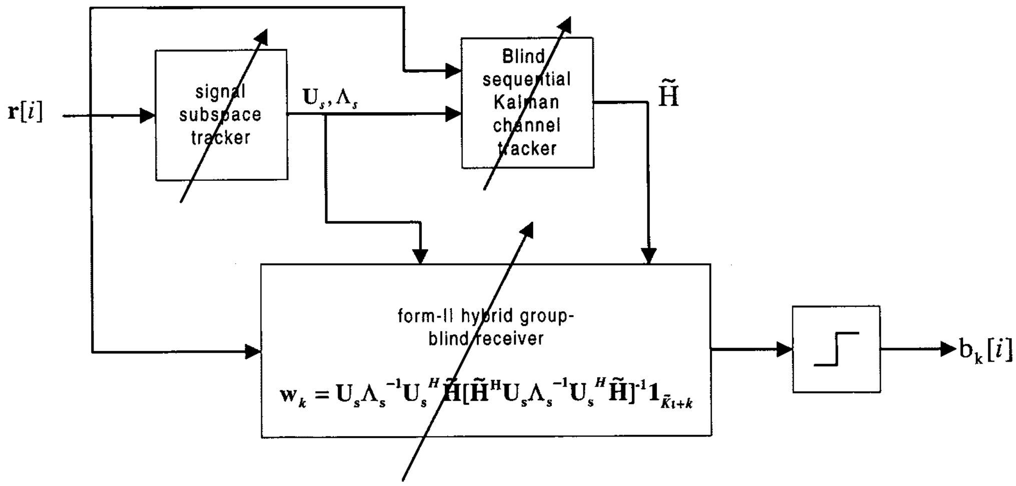

Fig. 1. Adaptive receiver structure.

adopt the blind, sequential channel estimation technique developed by Wang and Poor in [4]. It produces estimates of the composite signature waveforms of the known users without the need of a noise subspace estimate. Note that there is an arbitrary phase ambiguity in the estimated channel state, which necessitates differential encoding and decoding of the transmitted data.

IV. Adaptation via Subspace Tracking

Since the form-II hybrid group-blind detector may be written in closed form as a function of the signal subspace components, one may use a suitable subspace tracking algorithm in conjunction with this detector and a channel estimator to form an *adaptive* detector that is able to track changes in the number of users and their composite signature waveforms. Fig. 1 contains a block diagram of such a receiver. The received signal $\mathbf{r}[i]$ is fed into a subspace tracker which sequentially estimates the signal subspace components $(\mathbf{U}_s, \mathbf{\Lambda}_s)$. These estimates, along with the received signal, are fed to the blind sequential Kalman channel estimator developed in [4]. The linear hybrid group-blind detector (or the linear blind MMSE detector) is then constructed form the channel state estimate and the signal subspace component estimates.

Subspace trackers of various complexities and performance characteristics have appeared in the literature. In particular, the NA-CSVD [5] tracking algorithm was used successfully by Yu and Høst-Madsen for subspace tracking for group-blind multiuser detection over synchronous CDMA channels [6].

A. QR-Jacobi Methods

QR-Jacobi methods constitute a family of SVD-based subspace tracking algorithms that rely extensively on Givens rotations during the updating process. This reduces complexity and has the advantage of maintaining the orthonormality of matrices. Members of this family include NA-CSVD, RO-FST [7], NASVD [8], and the algorithm developed by Moonen *et al.* in [9].

Let $\mathbf{R}(l) = [\mathbf{r}[1]\cdots\mathbf{r}[l]]$ denote a $Pm \times l$ matrix whose columns contain the first l snapshots of the received signal. Define the matrix $\mathbf{\Gamma}(l) = \mathrm{diag}(\sqrt{\gamma}^{l-1}, \ldots, \sqrt{\gamma}, 1)$. Then the exponentially windowed sample correlation matrix is given by

$$\mathbf{C}(l) = \left(\frac{1}{M(l)}\right)\mathbf{R}(l)\mathbf{\Gamma}(l)\cdot[\mathbf{R}(l)\mathbf{\Gamma}(l)]^H \tag{20}$$

where $M(l) = (1-\gamma^l)/(1-\gamma)$ is the effective window length. Generally speaking, SVD-based subspace tracking algorithms attempt to track the SVD of a data matrix of growing dimension, defined recursively as

$$\mathbf{\Gamma}(l+1)\mathbf{R}^H(l+1) = \begin{bmatrix}\sqrt{\gamma}\mathbf{\Gamma}(l)\mathbf{R}^H(l)\\ \mathbf{r}^H[l+1]\end{bmatrix}. \tag{21}$$

We may write the SVD of this matrix as

$$\mathbf{\Gamma}(l+1)\mathbf{R}^H(l+1) = \mathbf{U}(l+1)\mathbf{\Sigma}(l+1)\mathbf{V}^H(l+1) \tag{22}$$

$$= \mathbf{U}(l+1)\begin{bmatrix}\mathbf{\Sigma}_s(l+1) & 0\\ 0 & \mathbf{\Sigma}_n(l+1)\end{bmatrix}\begin{bmatrix}\mathbf{V}_s^H(l+1)\\ \mathbf{V}_n^H(l+1)\end{bmatrix} \tag{23}$$

where $\mathbf{V}_s(l)$ is a matrix whose columns are eigenvectors that span the signal subspace and $\mathrm{diag}[\mathbf{\Sigma}_s(l)]$ contains the square-root of the corresponding eigenvalues. The matrix $\mathbf{U}(l)$ need not be tracked. Furthermore, since the noise subspace does not need to be calculated for the algorithm used in this letter, we do not need to track $\mathbf{V}_n(l)$ or $\mathbf{\Sigma}_n(l)$. This allows us to reduce complexity using noise averaging [10]. Since calculating the SVD from scratch at each iteration is time consuming and expensive, the issue then is how best to use the new measurement vector, $\mathbf{r}[l+1]$, to update the decomposition in (23).

Noise-averaged QR-Jacobi algorithms begin with a Householder transformation that rotates the noise eigenvectors such that the projection of the new measurement vector $\mathbf{r}[l+1]$ onto the noise subspace is parallel to the first noise vector, which we denote by $\mathbf{v_n}$. Specifically, let

$$\mathbf{r}_s = \mathbf{V}_s(l)^H\mathbf{r}[l+1] \tag{24}$$

$$\mathbf{V}_n = \frac{\mathbf{r}[l+1] - \mathbf{V}_s(l)\mathbf{r}_s}{\beta} \tag{25}$$

where $\beta = ||\mathbf{r}[l+1] - \mathbf{V}_s(l)\mathbf{r}_s||$. Then we may write the modified factorization

$$\begin{bmatrix} \sqrt{\gamma}\mathbf{\Gamma}(l)\mathbf{R}^H(l) \\ \mathbf{r}^H[l+1] \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{U}(l) & 0 \\ \hline 0 & 1 \end{array}\right] \cdot \left[\begin{array}{ccc} & \sqrt{\gamma}\mathbf{\Sigma}(l) & \\ \hline \mathbf{r}_s^H & \beta & 0 \end{array}\right] \cdot \left[\mathbf{V}_s(l)|\mathbf{V}_n|\mathbf{V}_n^{\perp}\right]^H \tag{26}$$

where $\mathbf{V}_n^{\perp}$ represents the subspace of $\mathbf{V}_n(l)$ that is orthogonal to $\mathbf{V}_n$. The second step in QR-Jacobi methods, sometimes called the *QR step*, involves the use of Givens rotations to zero each entry of the measurement vector's projection on the signal subspace. We refer the reader to [11] for details concerning the use of Givens matrices for this purpose. The QR step replaces the last row in the middle matrix in the decomposition in (26) with zeros. These are row-type transformations involving premultiplication of the middle matrix with a sequence of orthogonal matrices. We do not need to accumulate these transformations in $\mathbf{U}(l)$ since $\mathbf{U}(l)$ does not need to be tracked.

The next step, diagonalization step, involves at least one set each of column-type and row-type rotations to further concentrate the energy in the middle matrix along its diagonal. Sometimes called the *refinement* step, this is where many of the existing algorithms begin to diverge. The RO-FST algorithm, for example, performs two fixed sets of rotations in the diagonalization step but leaves the middle matrix in upper triangular form and does not attempt a diagonalization. This is particularly efficient for applications that do not require a full set of eigenvalues, but is not useful here since the signal subspace eigenvalues are required for the construction of the detector. The NA-CSVD algorithm, on the other hand, attempts to optimize the choice of rotations to achieve the best diagonalization possible. The algorithm we present in the next section achieves near-optimal results (for noise-averaged trackers) while incurring a computational burden that is smaller than that of NA-CSVD.

B. NAHJ-FST Subspace Tracking

The algorithm we present here is a member of the QR-Jacobi family in the sense that it uses Givens rotations during the updating process. However, this algorithm avoids the QR step entirely. Instead of working with the SVD-type decomposition in (22), we work with a decomposition of the form

$$\mathbf{C}'(l) = \mathbf{V}(l)\mathbf{\Sigma}^2(l)\mathbf{V}^H(l) \tag{27}$$

where $\mathbf{C}'(l) = M(l)\mathbf{C}(l)$ and $\mathbf{\Sigma}^2(l)$ is Hermitian and almost diagonal. This is simply an eigendecomposition except that we have relaxed the assumption that $\mathbf{\Sigma}^2(l)$ is perfectly diagonal. At each iteration, we use a Householder transformation and a vector outer product to update $\mathbf{\Sigma}^2(l)$ directly. We then use single set of two-sided Givens rotations to partially diagonalize the resulting Hermitian matrix. There is no need for a separate QR-step. Essentially, the diagonalization process used in this algorithm is a partial implementation of the well-known symmetric Jacobi SVD algorithm [11] (not to be confused with the family of QR-Jacobi *update* algorithms). This algorithm is

TABLE I
NAHJ-FST SUBSPACE TRACKING ALGORITHM

Given: $\mathbf{\Sigma}^2(l-1) = \begin{bmatrix} \mathbf{\Sigma}_s^2(l-1) & 0 \\ 0 & \overline{\sigma}^2(l-1)\mathbf{I} \end{bmatrix}, \mathbf{V}_s(l-1)$

1. Calculate $\mathbf{r}_s, \mathbf{v}_n$, and β according to (24) and (25).
2. Dropping the indices, generate the modified factorization
$$[\mathbf{V}_s|\mathbf{v}_n|\mathbf{V}_n^{\perp}] \left(\begin{bmatrix} \gamma\mathbf{\Sigma}_s^2 & & \mathbf{0} \\ & \gamma\overline{\sigma}^2 & \\ \mathbf{0} & & \overline{\sigma}^2\mathbf{I} \end{bmatrix} + \begin{bmatrix} [\mathbf{r}_s^H|\beta]^H[\mathbf{r}_s^H|\beta] & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \right) [\mathbf{V}_s|\mathbf{v}_n|\mathbf{V}_n^{\perp}]^H.$$
3. Let $\mathbf{R}_s$ be the $r+2$ principal submatrix of the matrix sum in step 2. Apply a sequence of $r+1$ Givens rotations to $\mathbf{R}_s$ to produce $\mathbf{R}_a = \mathbf{\Theta}_{r+1}^T \cdots \mathbf{\Theta}_1^T \mathbf{R}_s \mathbf{\Theta}_1 \cdots \mathbf{\Theta}_{r+1}$.
4. Let $\mathbf{\Lambda}_s$ be the diagonal matrix whose diagonal is equal to the first r elements of the diagonal of $\mathbf{R}_a$.
5. Let $\mathbf{U}_s$ be composed of the first r columns of $[\mathbf{V}_s|\mathbf{v}_n]\,\mathbf{\Theta}_1 \ldots \mathbf{\Theta}_{r+1}$.
6. Set $\mathbf{\Sigma}_s^2(l)$ equal to the $r+1$ principal submatrix of $\mathbf{R}_a$.
7. Let $\mathbf{V}_s(l)$ be composed of the first $r+1$ columns of $[\mathbf{V}_s|\mathbf{v}_n]\,\mathbf{\Theta}_1 \ldots \mathbf{\Theta}_{r+1}$.
8. Reaverage the noise power: $\overline{\sigma}^2(l) = \frac{(Pm-r-2)(\sqrt{\gamma\overline{\sigma}^2(l-1)})+|\hat{\sigma}^2|}{Pm-r-1}$ where $\hat{\sigma}^2 = (\mathbf{R}_a)_{r+2,r+2}$.

used to find the eigenstructure of a general fixed symmetric matrix and is known to generate more accurate eigenvalues and eigenvectors than the symmetric QR SVD algorithm, but with a higher computational complexity [12]. However, we do not perform the full sweep of $r(r-1)/2$ rotations required for the symmetric Jacobi algorithm, but only a carefully selected set of about r rotations. This is sufficient because the matrix that we wish to diagonalize already has much of its energy concentrated along the diagonal. This is a situation that the Jacobi algorithm can take advantage of but which the QR algorithm cannot. The Jacobi algorithm also has an inherent parallelism which the QR algorithm does not. Table I contains a summary of this algorithm, which we term NAHJ-FST for noise-averaged Hermitian–Jacobi fast subspace tracking.

1) The Algorithm: The first step in NAHJ-FST is the Householder transformation mentioned previously. The second step involves generating a modified factorization that maintains the equality $\mathbf{V}(l)\mathbf{\Sigma}^2(l)\mathbf{V}^H(l) = \gamma\mathbf{V}(l-1)\mathbf{\Sigma}^2(l-1)\mathbf{V}^H(l-1) + \mathbf{r}(l)\mathbf{r}^H(l)$. Step 3 requires that we apply $r+1$ Givens rotations in order to partially diagonalize $\mathbf{R}_s$. Ideally, we would apply these rotations to the off-diagonal elements that have the largest magnitude. However, since the off-diagonal maxima can be located anywhere in $\mathbf{R}_s$, finding the optimal set of rotations requires an $O(r^2)$ search for each rotation. This leads to an $O(r^3)$ complexity algorithm. In order to maintain low complexity, we have implemented a suboptimal alternative that is simple yet effective. Let $\mathbf{z} = [\mathbf{r}_s^H|\beta]^H$ be the vector whose outer product is used in the modified factorization of step 2. Suppose i_0, $1 \leq i_0 \leq r+2$ is the index of the element in $\mathbf{z}$ that has the largest magnitude. The set of elements we choose to annihilate with the Givens rotations is given by $\{(\mathbf{R}_s)_{i_0,j}\}_{j=1}^{r+2}$, $j \neq i_0$, where $(\mathbf{R}_s)_{i_0,j}$ represents the element on the i_0th row and jth column of $\mathbf{R}_s$. Of course if $(\mathbf{R}_s)_{i_0,j}$ is annihilated, so is $(\mathbf{R}_s)_{j,i_0}$. This choice of rotations is not optimal; in fact, since we retain the

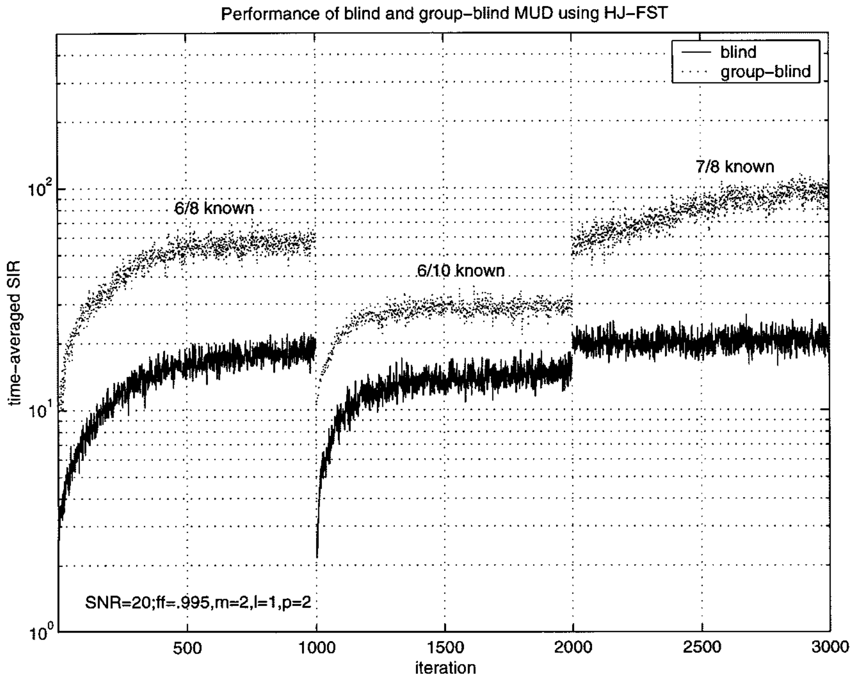

Fig. 2. Adaptive performance of NAHJ-FST.

off-diagonal information from the previous iteration, we cannot even be sure we annihilate the off-diagonal element in $\mathbf{R}_s$ with the largest magnitude. Nevertheless, we see that the technique is very simple and is somewhat heuristically pleasing. Ultimately, performance is the measure of merit and simulations show that it performs very well.

In order to adapt to changes in the size of the signal subspace (number of users), our tracking algorithm must be rank-adaptive. We have adopted the Akaike information criterion method. This method is often used in subspace tracking algorithms and is documented in [13]. In order to use this algorithm, we must track at least one extra eigenvalue/eigenvector pair. Hence, the appearance of $r+1$ in Table I.

2) Complexity: Complexity is a critical issue when considering subspace trackers for multiuser detection. Existing algorithms vary in complexity from $O(Pmr)$ to $O(P^3m^3)$. The NAHJ-FST algorithm has complexity $O(Pmr)$. In fact, when we consider trackers that deliver a complete set of signal subspace eigenvalues at each iteration, NAHJ-FST seems to have the lowest complexity of any algorithm that delivers comparable performance. Its nearest competitor appears to be NA-CSVD which has a complexity of $10Pmr+3Pm+7.5r^2+7r$ floating point operations (flops) per iteration using real data. NAHJ-FST requires approximately $3r^2$ fewer flops iteration. For the parameters used in our simulations, this results in about 17% fewer flops per iteration.

It is not the aim of this letter to present an exhaustive comparison of all of these QR Jacobi-type algorithms. It is sufficient to observe that NAHJ-FST has the lowest complexity of any algorithm that has been used for similar purposes, and that for the particular tracking problem we are addressing, the performance of the algorithm is close to the upper bound on the performance of all algorithms of this type. This upper bound is, at each iteration, given by an exact $O(P^3m^3)$ rank-one SVD update of the entire noise and signal subspaces. We present more details in the next section.

V. Simulation Results

In this section, we investigate the performance of our adaptive receiver in an asynchronous CDMA system. The processing gain $N=15$ and the spreading codes are Gold codes of length 15. The chip pulse waveform is a raised-cosine pulse with a roll-off factor of 0.5. The initial delay d_k of each user is uniform on $[0, 4T_c]$. Each user's channel has $L=3$ paths. The delay of each path $\tau_{k,l}$ is uniform on $[0, 6T_c]$. Hence, the maximum delay spread is one symbol interval, i.e., $\iota=1$. The fading gain of each path in each user's channel is generated from a complex Gaussian distribution and is fixed for all simulations. The path gains in each user's channel are normalized so that each user's signal arrives at the receiver with the same power. The oversampling factor is $p=2$ and the smoothing factor is $m=2$. Hence, this system can accommodate up to ten users [3]. The forgetting factor for the subspace tracking algorithms is 0.995.

The performance measures are bit-error probability and the signal-to-interference ratio (SIR) defined by $\text{SIR} \triangleq E^2\{\mathbf{w}^H\mathbf{r}\}/\text{var}\{\mathbf{w}^H\mathbf{r}\}$, where the expectation is with respect to the data bits of interfering users, the ISI bits, and the ambient noise. In the simulations, the expectation operation is replaced by the time averaging operation. The SIR is a particularly useful figure of merit for MMSE detectors since it has been shown [14] that the output of an MMSE detector is approximately Gaussian distributed. Hence, the SIR values translate directly and simply to bit-error probabilities.

Fig. 2 is a comparison of the adaptive performance of the MMSE blind detector and the hybrid group-blind detector using the NAHJ-FST subspace tracking algorithm. The signal-to-noise ratio (SNR) is fixed at 20 dB. During the first

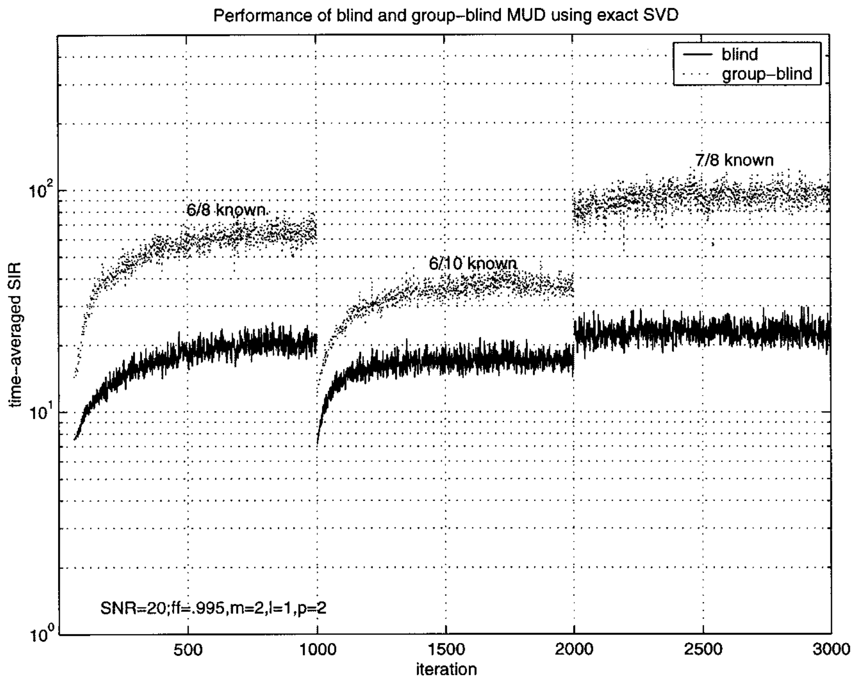

Fig. 3. Adaptive performance of exact SVD update.

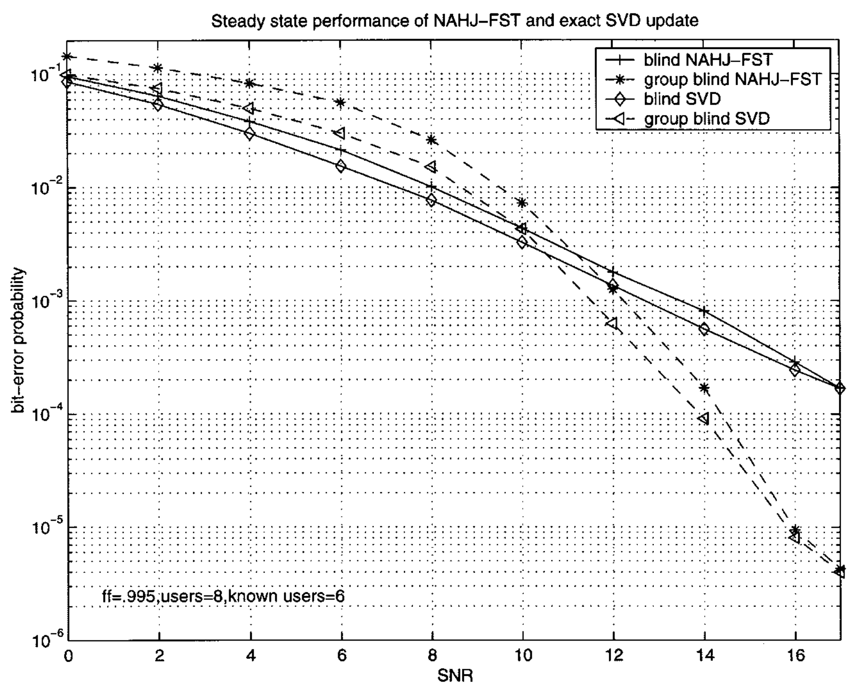

Fig. 4. Steady-state performance of NAHJ-FST and exact SVD.

1000 iterations, there are eight total users, six of which are known by the group-blind detector. At iteration 1000, two new users are added to the system. At iteration 2000, one additional known user is added and three unknown users vanish. We see that there is a substantial performance gain using the group-blind detector at each stage and that convergence occurs in less than 500 iterations.

Fig. 3 is created with parameters identical to Fig. 2 except that the tracking algorithm used is an exact rank-one SVD update. Again, we see a significant improvement in performance using the group-blind detector. More importantly, when we compare Figs. 2 and 3, we see very little difference between the performance we obtain using NAHJ-FST and that which we obtain using an exact SVD update.

Fig. 4 represents the steady-state bit-error rate (BER) performance of our receiver using NAHJ-FST and the exact SVD update for both blind and group blind multiuser detection. The number of users is 8 and the number of known users is 6. At SNRs above about 11 dB, we see that the group-blind detectors provide a substantial improvement in BER. At lower SNR, the group-blind detector seem to suffer from the noise enhancement problems that often accompany zero-forcing detectors. Recall that the hybrid group-blind detector zero forces the interference of the known users and suppresses the interference form the unknown users via the MMSE criterion. Once again, note the relatively small difference between the performance of NAHJ-FST and exact SVD, especially at high SNR.

VI. CONCLUSION

In this letter, we have developed a new low-complexity, high-performance, subspace tracking algorithm and applied it to an asynchronous multipath version of the group-blind multiuser detection problem proposed in [6]. We have seen that at moderate and high SNR, the adaptive hybrid group-blind detector provides a substantial performance gain over the blind adaptive MMSE detector. The noise enhancement problems of the hybrid group-blind detector at low SNR suggest the use of a dual-mode receiver that uses the blind approach for low SNR environments and the group-blind approach for moderate and high SNR environments. We have also demonstrated that the performance of NAHJ-FST in the context of adaptive multiuser detection is similar to the performance of an exact rank-one SVD update, which serves as a performance upper bound for all SVD-based subspace tracking algorithms.

REFERENCES

[1] H. V. Poor and S. Verdú, "Blind multiuser detection: A subspace approach," *IEEE Trans. Inform. Theory*, vol. 44, pp. 677–690, Mar. 1998.

[2] X. Wang and H. V. Poor, "Blind equalization and multiuser detection in dispersive CDMA channels," *IEEE Trans. Commun.*, vol. 46, pp. 91–103, Jan. 1998.

[3] X. Wang and A. Høst-Madsen, "Group-blind multiuser detection for uplink CDMA," *IEEE J. Select. Areas Commun.*, vol. 17, pp. 1971–1984, Nov. 1999.

[4] H. V. Poor and X. Wang, "Blind adaptive joint suppression of MAI and ISI in dispersive CDMA channels," in *Conf. Rec. Asilomar Conf. Signals, Systems, and Computers*, vol. 2, Nov. 1997, pp. 1013–1017.

[5] P. A. Pango and B. Champagne, "Accurate subspace tracking algorithms based on cross-space properties," in *Proc. Int. Conf. Acoustics, Speech, and Signal Processing*, vol. 5, Munich, Germany, 1997, pp. 3833–3836.

[6] J. Yu and A. Høst-Madsen, "Subspace tracking for group-blind multiuser detectors," in *Proc. IEEE Vehicular Technology Conf. (VTC99)*, Houston, TX, May 1999, pp. 1042–1046.

[7] D. J. Rabideau, "Fast, rank-adaptive subspace tracking," *IEEE Trans. Signal Processing*, vol. 44, pp. 2229–2244, Sept. 1996.

[8] A. Kavcic and B. Yang, "A new efficient subspace tracking algorithm based on singular value decomposition," in *Proc. 1994 IEEE Int. Conf. Acoustics, Speech, and Signal Processing*, vol. IV, 1994, pp. IV/485–IV/488.

[9] M. Moonen, V. Dooren, and J. Vandewalle, "A singular value decomposition algorithm for subspace tracking," *SIAM, Matrix Anal. Appl.*, vol. 13, no. 4, pp. 1015–1038, Oct. 1992.

[10] I. Karasalo, "Estimating the covariance matrix by signal subspace averaging," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-34, pp. 8–12, Feb. 1986.

[11] S. Haykin, *Adaptive Filter Theory*, 3rd ed. Englewood Cliffs, NJ: Prentice-Hall, 1996, pp. 536–560.

[12] R. Mathias, "Accurate eigensystem computations by Jacobi methods," *SIAM, Matrix Anal. Appl.*, vol. 16, no. 3, pp. 977–1003, July 1995.

[13] M. Wax and T. Kailath, "Detection of signals by information theoretic criteria," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-33, pp. 387–392, Apr. 1985.

[14] H. V. Poor and S. Verdú, "Probability of error in MMSE multiuser detection," *IEEE Trans. Inform. Theory*, vol. 43, pp. 868–871, May 1997.

[15] B. Yang, "Projection approximation subspace tracking," *IEEE Trans. Signal Processing*, vol. 44, pp. 95–107, Jan. 1995.